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# [Conjugate Sequence of Conjugate Partition Graph]

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## Abstract:

In this paper we will illustrate An asymptotic degree sequence , Also compute the conjugate degree sequence of random graph, Then we will illustrate a matrix for conjugate partition graph.

## 1. Introduction:

In graph theory , let G = (V , E) be a graph with vertex set V = $\{v_1 , v_2 ,\ldots.v_n \}$ , the degree of $v\epsilon V$ , denote by d(v) .

The degree sequence D = d(G) = $\{ d_1 , d_2 , d_3 \ldots\ldots d_n \}$ , where

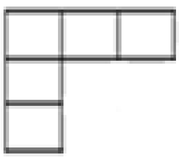
$d_1\geq d_2\geq \ldots\ldots\geq d_n \geq 0$ are the degrees of the vertices of G .

## 2. Definitions:

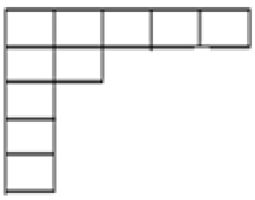
### 2.1 Self Conjugate:

A partition whose conjugate partition is equivalent to itself.

**As example:**



$\pi$ , $\pi^* = (3 ,1 ,1 ,0 )$



$\pi \cdot \pi^* (5 ,2 ,1 ,1 ,1)$

### 2.2 An asymptotic sequence is :

1. A sequence of integer value functions

$D = d_0(n)$ , $d_1(n)$ , …….such that :

a. $d_i(n) = 0$ if $I \geq n$.

b. $\sum_{i\geq 0} d_i(n) = n$.

2. D is a feasible if $\Omega_{Dn} \neq \phi$ for all $n \geq 1$.

3. D is smooth if there exist a constants $\lambda_i$ such that $\lim_{n\to\infty} d_i / n = \lambda_i$.

$\forall \epsilon > 0\ \exists N$ s.t $\forall n > N$ and $\forall i \geq 0$ then:

$$\left| i(i-2)\, d_i(n) / n - i(i-2)\lambda_i \right| < \epsilon$$

### 3. Main results:

**Definition 3.1:** let $d^*_j = o(\{ I , d_i \geq j \})$ , then the conjugate degree sequence ( CDS) $d^*(G) = (d_1^*, d_2^*, \ldots\ldots d_k^*)$ .

**Theorem 3.2 :**

For a graph G and degree sequence d(G) ,

Then $D^* = d^*(G)$ majorizes $D = d(G)$ .

**Lemma 3.3 :**

A conjugate degree sequence (CDS) is A sequence

$D^* = d_0^*(n)$ , $d_1^*(n)$ , …….s.t:

a. $d_i^*(n) = 0$ if $i \geq n$.

b. $\sum_{i\geq 0} d_i^*(n) = n$.

**proof:**

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$D^* = d_0^*(n)$ , $d_1^*(n)$ , …. $\forall$ i $\geq$ n , $d_i(n) = 0$ , then if

$d_i^*$ (n) = o ({ I , $d_i \geq$ I )} we conclude that $d_i^*$ (n) = 0 ,

$\sum_{i\geq 0} d_i^*(n) = \sum_{i\geq 0} d_i(n) = n$ .

**Lemma 3.4:**

A conjugate degree sequence is a feasible if $\Omega_{Dn}^* \neq \phi$ for all $n \geq 1$.

**Proof:**

If if $\Omega_{Dn} \neq \phi$ and $D^* \geq D$ for $n \geq 1$

Then $\Omega_{Dn}^* \neq \phi$ .

**Lemma 3.5:**

A conjugate degree sequence is smooth if there exist a constants $\mu_i$ such that $\lim_{n\to\infty} d_i^* / n = \mu_i$.

**Proof:**

From theorem 3.2 , $D^* = d^*(G)$ majorizes D and if $\lim_{n\to\infty} d_i / n = \lambda_i$

Then there exist $\mu_i$ s.t $\lim_{n\to\infty} d_i / n = \mu_i$

Where $\mu_i$ is constant different from $\lambda_i$ .

**Lemma 3.6:**

As asymptotic degree sequence , CDS

$D^*$ is well or the best if:

a. $D^*$ is feasible .

b. $j(j-2)\, d_j^*(n) / n$ goes uniformaly to $j(j-2)\, \mu_i$ .

$\forall\, \epsilon > 0\ \exists\, N$ s.t $\forall\, n > N$ and $\forall\, j \geq 0$ then:

$\left| j(j-2)\, d_j^*(n) / n - j(j-2)\, \mu_i \right| < \epsilon$.

**Theorem 3.7:**

Let $G^{*} = ( V, E )$ , be a conjugate of graph with set $\{ v_1 , v_2, v_3 ,\ldots\ldots v_n \}$ , if $O(E) = m$ , then:

1. $\sum_{i\geq0}^{n} d^{*} \geq 2m$ if $G^{*} \geq G$.

2. . $\sum_{i\geq0}^{n} d^{*} = 2m$ if G is a self conjugate graph.

**Proof:**

1. Since $D^{*} = d^{*}(G)$ majorizes $D= d(G)$ , if $O(E) = m$ ,

$d_j^{*} = O (\{ d_i , d_i \geq j \}) , j = 1,2,\ldots$

the sequence $d_1^{*} , d_2^{*} ,\ldots\ldots$ is said to be conjugate to the sequence

$d_1 , d_2 , \ldots\ldots d_n$ , we have $d^{*}(v_1) \geq d (v_1)$

$d^{*}(v_1) + d^{*}(v_2) \geq d (v_1) + d (v_2)$.

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$d^{*}(v_1) +\ldots\ldots + d^{*}(v_n)\geq d (v_1) + \ldots. + d (v_n)$

$\sum_{i\geq0}^{n} d^{*} ( v_i ) \geq \sum_{i\geq0}^{n} d (v_i) = 2m.$

$\sum_{i\geq0}^{n} d^{*} ( v_i ) \geq 2m.$

2. If G is self conjugate graph then we have:

$d_1 = d_1^{*}$

$d_2 = d_2^{*}$

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$d_n = d_n^{*}$

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Then $\sum_{i\geq 0}^{n} d^{*}(v_i) \geq \sum_{i\geq 0}^{n} d(v_i) = 2m$.

**Example1:**

For graph shown in Fig.(1. a ) ,the conjugate graph is obtained from g by replacing rows by column.

$\pi = (6, 3^2, 2, 1)$, then $\pi^* = (5, 4, 3, 1^3)$.

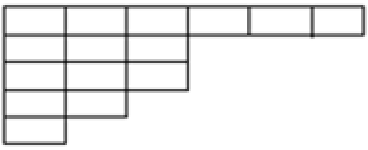


Fig.(1.a)

$\pi$

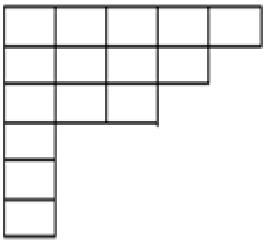
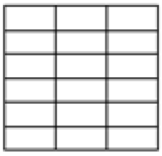


Fig. (1. b)

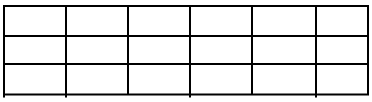
$\pi^*$

**Example2:**

If $\pi = (3^6)$ then $\pi^* = (6^3)$ .



$\pi$



$\pi^*$

**3.8 Writing conjugate partitions as a matrix:**

We will write the conjugate partitions( as figured before) as a matrix

The conjugate matrix for partitions will take constant form and changed from the origin matrix for partition as follows:

If the partition matrix take the form of:

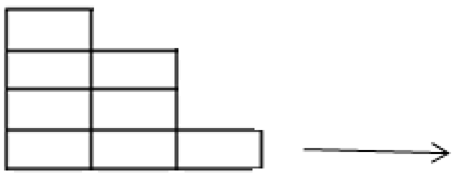
$$\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & \cdots & & \cdots \\ \cdots & \cdots & & \cdots \\ \cdots & & \cdots & \cdots \\ \cdots & & \cdots & \cdots \\ a_{n1} & \cdots & \cdots & a_{nn} \end{vmatrix}$$

Then the conjugate partition matrix will take the form:

$$\begin{vmatrix} a_{n1} & a_{(n-1)1} & \cdots & a_{11} \\ & & \cdots & \\ \cdots & \cdots & & \cdots \\ \cdots & \cdots & & \cdots \\ \cdots & & \cdots & \cdots \\ \cdots & & \cdots & \cdots \\ a_{nn} & \cdots & \cdots & a_{1n} \end{vmatrix}$$

**Example 3:**

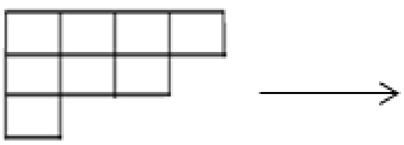
For partition graph shown in Fig. (2) ,



$\pi$

$$\begin{vmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{vmatrix}$$

Then the conjugate partition matrix will be:



$\pi^*$

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{vmatrix}$$

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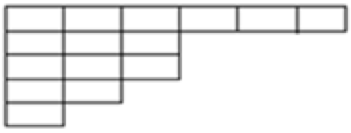
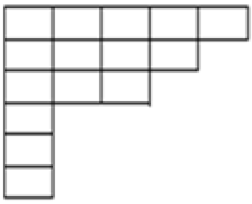
**Example 4:**

For partition graph shown in Fig.(1 ) the matrix can be written as:



$\pi$

$$\longrightarrow \left|\begin{array}{cccccc} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right|$$



$\pi^*$

$$\longrightarrow \left|\begin{array}{cccccc} 1 & 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{array}\right|$$

**Note that:**

The matrix of partition graph or partition conjugate graph must have the same size.

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